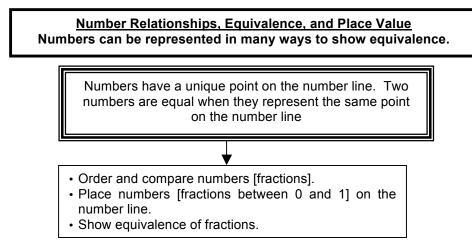
CONCEPT LESSON OVERVIEW

Student Task:

In this lesson, students work with two different unit wholes and fractional parts of a whole. They will position and compare fractions on a number line and determine equivalence of fractions on a number line.



Mathematical Goals:

- deepen students' understanding of fraction sense in terms of the meaning of numerator, denominator, whole, part of the whole, and equivalence
- deepen students' understanding of fractions as numbers, beginning with unit fractions, by building on the idea of partitioning a whole into equal parts.
- develop an understanding of unit fractions as the basic building blocks of fractions in the same sense that 1 is the building block of whole numbers
- develop an understanding of the relative positions of fractions between 0 and 1 on a number line
- develop an understanding of the equivalence of fractions
- develop an understanding of comparing fractions by reasoning about their size
- develop an understanding of comparing fractions with same numerators

Materials:

- · Tasks (attached)
- · Recording sheets with number lines (attached)
- Strips of paper representing the different types of bars (attached)
- Optional colored pencils

Los Angeles Unified School District revised 10.31.13

Common Core Standards Addressed in the Lesson:

Cluster: Develop understanding of fractions as numbers.

- **3.NF.1** Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a/b* as the quantity formed by a parts of size 1/*b*.
- **3.NF.2** Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal pars. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.

b. Represent a fraction *a/b* on a number line diagram by marking off a lengths 1/*b* from 0. Recognize that the resulting interval has size *a/b* and that its endpoint locates the number *a/b* on the number line.

3.NF.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Math Practice 1 (MP1): Math Practice 2 (MP2):	Make sense of problems and persevere in solving them. Reason abstractly and quantitatively.
Math Practice 3 (MP3):	Construct viable arguments and critique the reasoning of others.
Math Practice 4 (MP4):	Model with mathematics.
Math Practice 5 (MP5):	Use appropriate tools strategically.
Math Practice 6 (MP6):	Attend to precision.
Math Practice 7 (MP7):	Look for and make use of structure.
Math Practice 8 (MP8):	Look for and express regularity in repeated reasoning.

Language Goals:

Students will ask clarifying questions to contextualize the problem.

Students will justify their answers by using pictures, words, or other representations. Students will orally explain the sequence to arriving at a mathematical solution.

Academic Language:

The concepts represented by these terms should be reinforced/developed through the lesson:

- Number
- Fractions
- Unit fractions
- Number line
- Unit
- (Unit) Whole
- Equivalent/Not equivalent
- Numerator
- Denominator
- Part (of the whole)

Encourage students to use multiple representations (drawings, manipulatives, diagrams, words, number(s)), to explain their thinking.

Assumption of prior knowledge/experience:

Understand fractions as parts of a whole or unit Use of number lines Experience folding paper in halves, fourths ("Fraction Kit" lesson in *About Teaching Mathematics*, by Marilyn Burns)

Organization of Lesson Plan:

- The left column of the lesson plan describes rationale for particular teacher questions or why particular mathematical ideas are important to address in the lesson and how standards for mathematical practice are incorporated.
- The right column of the lesson plan describes suggested teacher actions and possible student responses.

<u>Key</u>:

Suggested teacher questions are shown in bold print. Standards for Mathematical Practice are marked with MP and their number.

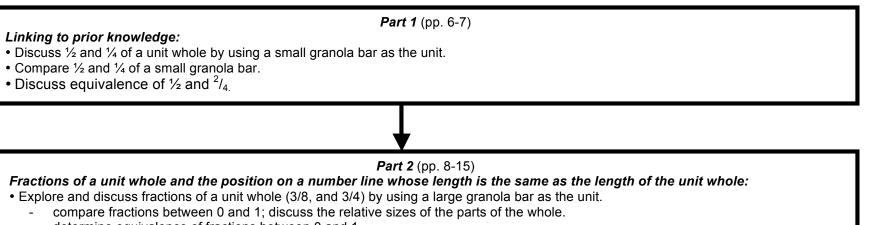
Possible student responses are shown in italics.

** indicates key questions in terms of the goals of the lesson.

Lesson Phases:

The phase of the lesson is noted on the left side of each page.

THE LESSON AT A GLANCE



- determine equivalence of fractions between 0 and 1.
- locate and compare fractions between 0 and 1 on a number line whose length is the same length as the granola bar.



- Fractions are equal shares or equal-sized parts of a whole. The denominator of a fraction tells you the number of equal parts into which the whole is divided. The more fractional parts used to make a whole, the smaller each of the parts will be.
- Fractions are built out of unit fractions, just as whole numbers are composed of tens and ones.
- Equivalent fractions describe the same amount by using different sized parts.
- Two fractions are equivalent when they represent the same point on the number line.
- Comparing fractions with the same numerator can be done by reasoning about the size of the unit fraction.

THE LESSON

Phase	RATIONALE	SUGGESTED TEACHER QUESTIONS/ACTIONS AND POSSIBLE STUDENT RESPONSES
S	HOW DO YOU SET UP THE TASK?	HOW DO YOU SET UP THE TASK?
E T U P S E T U P S E	 Solving the task prior to the lesson is critical so that you: become familiar with strategies students may use. consider the misconceptions students may have or errors they might make. This will allow you to better understand students' thinking and prepare for questions they may have or that you might ask. It is important that students have access to solving the task from the beginning. The following strategies can be useful in providing such access: strategically pairing students who complement each other. providing manipulatives or other concrete materials. identifying and discussing vocabulary terms that may cause confusion. posting vocabulary terms on a word wall, including the definition and, when possible, a drawing or diagram. It is important not to "teach" the terms prior to the lesson. Instead, use the word wall as a tool to assist students if and when they encounter difficulty with a term. 	 Solve the task in as many ways as possible prior to the lesson. Make certain students have access to solving the task from the beginning by: having students work with a partner. having paper strips representing the granola bar on each student's desk. having the granola bar displayed on an overhead projector or black board so that it can be referred to as the problem is read. making certain that students understand the vocabulary used in the task (i.e. part, whole, amount, numerator, denominator, equivalent, number line). The terms that may cause confusion to students could be posted on a word wall and referred to if and when a confusion arises.
T U P	SETTING THE CONTEXT FOR THE TASK Linking to Prior Knowledge It is important that the task have points of entry for students. By connecting the content of the task to previous mathematical knowledge, students will begin to make the connections between what they already know and what we want them to learn. (MP4)	SETTING THE CONTEXT FOR THE TASK Think about a time when you had to share some food. How did you know if your parts were fair? (Think-Pair-Share)

Phase	RATIONALE	SUGGESTED TEACHER QUESTIONS/ACTIONS AND POSSIBLE STUDENT RESPONSES
S E	<u>PART 1</u>	PART 1
т	 Prepare student materials by cutting out one Play Day bar per student. Using models or concrete objects when discussing fractions helps students clarify ideas that would 	Now let's think about sharing granola bars: • You and your partner each have a "Play Day" granola
U P	otherwise confuse them when only using symbolic notation. (MP4; MP5)	bar at your desks. (Hold up the strips representing the bars for students to see.)
	 Allowing students to construct viable arguments and critique the reasoning of other provides opportunities for student discourse and use of academic vocabulary. (MP3; MP6) 	
S E T	 Asking students to demonstrate using the strips of paper allows the teacher to get a sense of who has a grasp of the fractions ½ and ¼. (MP5) 	 Suppose one student can only have ½ of a Play Day and another student can only have ¼ of a Play Day. Show me how much of a granola bar each student can
U P	 Be aware that some students may partition the bar by folding horizontally, or horizontally and vertically. Think about how that correct fractional representation might not be a match with the number line representation. 	 have. <u>Group Discussion Questions</u> How are you sure you got exactly ½ or ¼ of the bar?
S	<u>Group Discussion Questions</u> By estimating	(By estimating or by folding the paper) Have students demonstrate to the rest of the class how they estimated $\frac{1}{2}$ and $\frac{1}{4}$ of the bars.
E T	 Students may say they just estimated where half of the one-fourth of the bar would be. 	 How many people did it's way? What are other ways to show the amounts? Have a
U	By folding the strip of paper	student or students who folded the one strip in half and folded the other strip in half and then half again
	 Students may demonstrate by folding one strip in half and folding the other strip in half and then half again. 	demonstrate their solution. Ask them to explain where the $\frac{1}{2}$ and $\frac{1}{4}$ are and how they know they are correct.
	Part-Whole Language	Which ways are more precise? Why?
	 Use part-whole language. Make sure that you do not say things like "Here is a piece of granola bar" when you are referring to the whole. (A piece can be a part of the granola bar or one whole piece of granola bar.) Keeping the language clear during this lesson will help students make connections and understand the concept of fractional parts. (MP6) 	• So how many parts was the Play Day divided into for
		the person who got ½? How do you know? (2 parts since the denominator was 2)
		• How many parts was the Play Day divided into for the person who got 1/4? How do you know? (4 parts since the denominator was 4)

Phase	RATIONALE	SUGGESTED TEACHER QUESTIONS/ACTIONS AND POSSIBLE STUDENT RESPONSES
s	Comparing Fractions and Equivalent Fractions	Comparing Fractions and Equivalent Fractions
E T U	 Asking students consistently to explain how they know something is true develops in them a habit of explaining their thinking and reasoning. This leads to deeper understanding of mathematics concepts. (MP3) 	 **Which is more ½ or ¼ of a granola bar? How do you know? What if I wanted you and your partner to have the same amount of the bar? What would we need to do?
Ρ	 Asking students how to make both partners have the same amount of granola bar links to their prior knowledge of fractions and will help scaffold their learning during this lesson. (MP2) 	(We would need to give the person who has ¼ of a bar another ¼ OR take half of the ½ bar.) Have a student or pair of students come up and demonstrate using their paper strips.
S E T U P	 The question "Can we say that ½ = 2/4?" is being asked and written this way in order to draw students' attention to the equivalence between the two fractions. Just because students have increased the amount of the one bar to make it equal to the other bar does not necessarily mean that they understand that the two amounts are now equal. They also experience difficulty understanding how ½ can equal 2/4 when prior to this time larger numbers always meant larger quantities. (MP7) 	 **So can we say that ½ = 2/4? Why?
S E T	 Students are able to recognize a regular pattern between both numerator and denominator in a fraction, allowing for predictions of an infinite set of fractions that are equivalent. (MP8) 	 Summary Statement: So equivalent fractions are two ways of describing the same amount by using different sized parts.
U P	 Summarizing key mathematical points and beginning to make use of structure lets students know they have said or discovered something that is mathematically important to know. (MP7) Using talk moves like restating allows student to hear and say the same idea in many different ways, allowing for greater understanding. 	

Phase	RATIONALE	SUGGESTED TEACHER QUESTIONS/ACTIONS AND POSSIBLE STUDENT RESPONSES
S	PART 2	<u>PART 2</u>
E T U P	 Prepare the Star Bars by cutting two bars per student. Using models or concrete objects when discussing fractions helps students clarify ideas that would otherwise confuse them when only using symbolic notation. (MP4; MP5) 	• Now we are going to look at an even bigger granola bar. (Give each student <u>2 Star Bars</u> , the <u>Part 2 task sheet</u> , and the <u>recording sheet with number lines</u> whose lengths are equivalent to the length of the Star Bar.)
	• Asking students to state what they think they are trying to	Ask a student to read the task as others follow along: Think about this:
	find allows them to put in their own words what they will be doing. It also gives the teacher the opportunity to assess	John has $\frac{3}{6}$ of a Star Bar.
S	whether or not students understand the problem they are	Sue has ¾ of a Star Bar.
E T	to solve. (MP1)	Who has the largest part of a Star Bar? Use a model, numbers and words to justify your answer. Show your
U P		solution in at least two ways. Be prepared to explain how you figured out the part of the granola bar that each person received and how you know which person received the most.
		Ask students to think-pair-share: What do you know? What are you trying to find out? Then ask students to share out what they know and what they are trying to find out, encouraging them to share something a partner may have said. Use the talk move of restating as students share. Ask: What questions do you have?
	INDEPENDENT PROBLEM-SOLVING TIME	INDEPENDENT PROBLEM-SOLVING TIME
	It is important that students be given private think time to understand and make sense of the problem for themselves and to begin to solve the problem in a way that makes sense to them. (MP1)	 Tell students to work on the problem by themselves for at least five minutes. Circulate around the class as students work individually. Ask focusing, assessing and advancing questions, but do not tell them how to solve the problem.

Phase	RATIONALE	SUGGESTED TEACHER QUESTIONS/ACTIONS AND POSSIBLE STUDENT RESPONSES
E X	FACILITATING SMALL-GROUP EXPLORATION	FACILITATING SMALL-GROUP EXPLORATION
A P L O R E	What do I do if students have difficulty getting started? It is important to ask questions that do not give away the answer or that do not explicitly suggest a solution method. (MP1)	Tell students they may now work with their partners. As students continue working, circulate around the classroom. <u>What do I do if students have difficulty getting started?</u> Ask questions such as: • What are you trying to figure out? • Who do you think has the most? • How can you show their amounts?
	Possible misconceptions or errors:	Possible misconceptions or errors:
E X P L O R E	It is important to have students <u>explain their thinking</u> before assuming they are making an error or have a misconception. After listening to their thinking, <u>ask</u> <u>questions</u> that will move them toward understanding their misconception or error. (MP1; MP3)	 Eighths are larger than fourths since 8 is larger than 4. Show me on your paper strip how many parts each person's bar is divided into. So which is a larger part of a bar – an eighth or a fourth? How do you know? The amounts are the same, since each has 3 parts. Show me on your paper strips how many parts of the bar each student gets.
	 Having students demonstrate how they are solving the problem gives the teacher insight into how the students 	Making connections between the Play Day and Star Bar
Е	 <u>Revoicing</u> a contribution marks that the contribution is <u>important mathematically</u>. It also <u>makes public</u> one student's thinking from which other students may learn and provides opportunities for communication. (MP3) 	• We found ½ of the Play Day bar earlier. How could we find ½ of the Star Bar? How might knowing where ½ is help us know the size of the two fractions we're comparing?
X P		Most students will fold the paper strip in half. I made two parts and $\frac{1}{2}$ is one of the parts.
L O R E		Ask students to explain how they know. They should be able to state that the denominator of the fraction tells how many equal parts the bar is divided into and the numerator tells how many of the equal parts are counted.
		 Revoice the student's contribution by saying: So there are two halves in the whole - two EQUAL parts – and ½ means we count one of them.

Phase	RATIONALE	SUGGESTED TEACHER QUESTIONS/ACTIONS AND POSSIBLE STUDENT RESPONSES
Е	Connecting to the number line	Connecting to the number line
X P L	• Since the number line is the same length as the paper strip, students will realize that half the paper strip can be represented by a point halfway between 0 and 1. (MP5)	 Connecting to the number line. Now look at the number line. How could we show ½ on the number line?
O R E		- If students are not familiar with the number line ask them How does the number line relate to your paper strip?
	Exploring Sue's part (3/4 of the Star Bar)	Exploring Sue's part (3/4 of the Star Bar)
	Remember to press students to explain how they know	Using the strip of paper:
	 something is true. (MP3; MP6) Asking students to explain other students' thinking is a way of assessing their understanding of the concept or uncovering possible misconceptions or errors. (MP3; MP6) Consider asking students to color in the portion of the bar, if they are having difficulty seeing the amount represented. (MP5) Students may fold the bar horizontally and vertically to create four equal pieces. Consider how that model might work with the number line. 	 How can we show Sue's part of the granola bar and explain how we know it is ³/₄ of the granola bar?
E X		I folded it into four parts and she gets three of the parts. (Have student demonstrate how she did this.)
P L O R E		 Revoice what the student said: So you folded it in half and then in half again. So there are four equal pieces. (Have student hold up the folded strip of paper.)
		 How did you know to fold the strip into four equal parts?
	 Students should be encouraged to share their solution 	The denominator of the fraction tells you how many parts.
	path with a partner to the extent that their partner can explain it to someone else. They should also be encouraged to make connections between their solution and their partner's. (MP3)	• Connecting to the number line. Now look at the number line. How could we show what part of the bar Sue got on the number line? Show me Sue's part of the bar using the strip.
E X P		-Students might fold their paper strips and line them up with the number line. Make certain that students talk about where $\frac{3}{4}$ would be.
L O R		- Students might fold the paper with the number line on it in the same way they folded their paper strips. Be certain they talk about where the ³ / ₄ would be.
E		- Students might talk about estimating where the ³ / ₄ would be using their knowledge of fractions.

Phase	RATIONALE	SUGGESTED TEACHER QUESTIONS/ACTIONS AND POSSIBLE STUDENT RESPONSES
Е		• How did you know that you needed 3 of the parts?
X P		The numerator of the fraction tells you how many of the parts you need.
L O R E		Summarizing the students' contributions: So the denominator told you how many parts to divide the granola bar into, and the numerator told you how many of the parts you need.
E		Have students display this on a projection system. If students do not suggest either of the solution paths above then say you heard someone talk about this method and present it. Ask other students if they understand it, if someone can put it into their own words, or if someone can add on.
X	Equivalence	<u>Equivalence</u>
P L O	High-level tasks such as this one provide the opportunity to make use of structure and develop deeper understanding of other important and related mathematical concepts. (MP7)	Hold up Sue's bar to the number line. What part of Sue's bar is equivalent to ½ on the number line? What would you call it? How do you know?
R E	 Determining ³/₄ both by adding 2/4 + ¹/₄ and by using the paper strips will allow students to make connections between the concrete model and the symbolic notation. (MP4) 	How much greater is Sue's bar than the ½? Ask a student to show this with the paper strip
	Comparing Fractions	Comparing Fractions
	Students should now be able to move from using the paper strips to using the number line to locate fractional parts. (MP2; MP5)	<u>Comparing Fractions</u> Look at the number lines for John and Sue. Who got the largest part of a Star Bar? How do you know?
	 Looking at the solution in more than one way will deepen students' conceptual understanding and help them check and see if their answers make sense. (MP2) When comparing two whole numbers on a number line, the one on the right is always larger, and the one on the left is said to be smaller. The same applies to fractions. 	• Connecting to the number line. Now look at the number line. How could we show what part of the bar we got on the number line? Show me your part of the bar using the strip. Where would that amount be on the number line?

Phase	RATIONALE	SUGGESTED TEACHER QUESTIONS/ACTIONS AND POSSIBLE STUDENT RESPONSES
E X P L O R E	Comparing Fractions Using the Number Line By having 2 number lines on the same paper and having the 0's and 1's aligned, it will be easier for students to make the connection between folding the paper strips and locating fractions on the number line. (MP5) The number line reinforces the analogy between fractions and whole numbers, many fractions can label the same point on the number line. Equivalence is said to occur when two fractions name the same point on the number line.	 <u>Comparing Fractions Using the Number Line</u> Let's look at the number lines. How can we tell by looking at the number lines who has the largest part of a Star Bar? What if we didn't have our paper strips – how could we find those fractions on a number line? Students should make the connection between folding the paper strips and folding the number line to locate fractions. Students might also estimate "halfway", "one fourth of the way", etc. on the number line.
E X P L O R E		

S	FACILITATING THE SHARE, DISCUSS, AND ANALYZE	FACILITATING THE SHARE, DISCUSS, AND ANALYZE
H A R	What solution paths will be shared, in what order, and why? The purpose of the discussion is to assist the teacher in	What solution paths will be shared, in what order, and why?
E	making certain that the goals of the lesson are achieved by the students. Questions and discussions should focus on	Possible Solutions to be Shared
D	the important mathematics and processes that were identified for the lesson.	<u>Comparing Halves</u> (half of the Play Day and half of the Star Bar)
Т		(Show the halves of each bar on the overhead)
S C	Mark for the students that this is the most important part of the lesson and that you expect them to listen attentively, ask	 **I am confused. How can both of these be a half? How can halves be different sizes?
U S	questions of each other, defend their reasoning, and make connections between others' solution paths and their own.	- The "smaller" half is half of a small granola bar. The "bigger" half is half of a large granola bar.
S	(MP3; MP7)	- It has to do with the size of the granola bar.
A	** Indicates questions that get at the key mathematical ideas in terms of the goals of the lesson	 **Summary: So it all depends on the size of the whole? How can we put in words that halves can be different?
N D	Possible Solutions to be Shared Comparing Halves	
	Playing "devil's advocate" allows students to see that the	Comparing Fractions
A N A	teacher has a question about something that students can help figure out. (MP3; MP6)	 **One group told me they thought that 3/8 was the largest part of a Star Bar since it is in eighths and 8 is bigger than 4. Do you agree? Why or why not?
L Y Z	• Stating an important idea and then having students state it in their own words allows the teacher to determine students' understanding. It also allows students to	-Students should state that eighths means the granola bar is divided into 8 equal parts while fourths means the bar is divided into four equal parts.
E	clarify their thinking about the concept and use the talk move of restating. (MP3; MP6))	 **Both of the numerators in ¾ and 3/8 are three. How can we compare fractions when they have the same numerator?
	 <u>Comparing Fractions</u> Beginning the discussion by asking students to agree or disagree allows them to think about and communicate why their answer is correct. (MP3; MP6) 	-Students should state that three of the smaller parts, eighths, is less than three of the larger parts, fourths.

Phase	RATIONALE	SUGGESTED TEACHER QUESTIONS/ACTIONS AND POSSIBLE STUDENT RESPONSES
S	FACILITATING THE SHARE, DISCUSS, AND ANALYZE	FACILITATING THE SHARE, DISCUSS, AND ANALYZE
H A R E	 <u>Comparing Fractions (continued)</u> This discussion addresses the possible misconception that the bigger denominator means a bigger number, an important opportunity for students to make use of the provide a statement of the statement of	<u>Comparing Fractions (continued)</u> The greater the denominator is, the more equal parts the whole is divided into. So each part would be smaller. You might ask a student to demonstrate this using the paper strip, or you might show it to the whole class.
D I S C U	structure and communicate it to others. (MP6; MP7) Using the Number Line to Answer the Question It would be helpful to display the number lines on the	 <u>Using the Number Line to Answer the Question</u> **Let's look at our number lines. How did you know where to put your fractions on the number line?
S S	 overhead so that all students can see them and refer to them during the discussion. (MP5) Press students to realize that the largest fraction in this 	Have several students give and show their explanations. - Students might talk about folding their paper strips and lining them up with the number line. Make certain that students talk about where fourths and eighths would be.
A N D	problem would be the one closest to 1 since it would be closest to a whole bar. (MP2; MP3)	 Students might talk about folding the paper with the number lines on it in the same way they folded their paper strips. Be certain to talk about and refer to the fourths and eighths on the number line. Students might talk about estimating where the fractions
A N A		 would be using their knowledge of fractions. **Who has the largest part of a Star Bar? How do you know by using the number line?
L Y Z E		 Students should state that ¾ is the largest part of the bar, because it is to the right of 3/8 on the number line. Some students might say that ¾ is larger because it is closer to 1. Some students analyze the size of a fraction by
		figuring out how far away it is from one whole. How far away from one whole is ³ / ₄ ? How far away from one whole is 3/8? When might this strategy be helpful?
		Students should be able to state that ³ / ₄ is only ¹ / ₄ away from the 1 whole, and 3/8 is 5/8 away from the whole. This strategy would be helpful when the numerator and denominator have a difference of 1.

PLAY DAY	PLAУ DAУ
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"Play Day" Granola Bars for Part 1 Activity



Fractions of a Granola Bar

Think about this problem:

- John has 3/8 of a Star Bar.
- Sue has ³/₄ of a Star Bar.

Who has the largest part of a Star Bar?
 Use a model, numbers, and words to justify your answer.
 Show your solution in at least two ways.



RECORDING SHEET FOR PART 2 ACTIVITY

Los Angeles Unified School District

Fractions of a Granola Bar - Recording Sheet

